CONSERVATIVE AND NON-CONSERVATIVE FORCES

Work, Power & Energy - Chapter Notes

CrackNeet Physics - NEET Physics Notes

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1. Introduction to Forces in Mechanics

In mechanics, forces are broadly classified based on how they do work on objects. Understanding this classification is crucial for mastering energy conservation concepts—a cornerstone topic for NEET Physics.

The Big Question: Does the path matter when a force does work?

This simple question divides all forces into two fundamental categories:

- **Conservative Forces** - Path doesn't matter
- **Non-Conservative Forces** - Path matters

This distinction determines whether mechanical energy is conserved in a system.

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2. Conservative Forces - Detailed Theory

2.1 What is a Conservative Force?

Definition: A force is said to be **conservative** if the work done by the force in moving an object from one point to another is **independent of the path taken** and depends only on the initial and final positions.

Alternative Definition: A force is conservative if the work done by it over any closed path is zero.

2.2 Key Characteristics of Conservative Forces

1. **Path Independence**
   - Work done depends only on initial and final positions
   - Different paths between same two points → same work done
   - Mathematical condition: $W_{AB}^{(path\ 1)} = W_{AB}^{(path\ 2)}$
2. **Closed Loop Property**
   - Work done in any closed path (returning to starting point) is zero
   - Mathematical condition: $\oint \vec{F} \cdot d\vec{r} = 0$
3. **Potential Energy Can Be Defined**
   - Conservative forces are associated with potential energy
   - Work done by conservative force = Negative of change in potential energy
   - $W_c = -\Delta U = -(U_f - U_i)$

4. **Mechanical Energy is Conserved**
   - When only conservative forces act, total mechanical energy remains constant
   - $E_{mechanical} = KE + PE = constant$

2.3 Physical Meaning - Why Path Doesn't Matter

Think of climbing a mountain:

Gravitational Force (Conservative):

- You can take a straight steep path or a winding gentle path
- The vertical height gained is the same
- Work done against gravity is the same: $W = mgh$ (height only)
- Path doesn't affect the work

Key Insight: Conservative forces depend only on position (initial and final), not on how you got there.

2.4 Examples of Conservative Forces

1. **Gravitational Force**
   - Universal gravitation between masses
   - Work done: $W = -mg(h_f - h_i) = -mg\Delta h$
   - Potential energy: $U = mgh$
2. **Elastic Spring Force**
   - Hooke's Law: $F = -kx$
   - Work done: $W = -\frac{1}{2}k(x_f^2 - x_i^2)$
   - Potential energy: $U = \frac{1}{2}kx^2$
3. **Electrostatic Force**
   - Coulomb force between charges
   - Work done depends only on separation between charges
   - Potential energy: $U = \frac{kq_1q_2}{r}$
4. **Magnetic Force (in some cases)**
   - Magnetic force due to magnetic poles

2.5 Mathematical Test for Conservative Force

A force $\vec{F}$ is conservative if:

$$\nabla \times \vec{F} = 0 \quad \text{(Curl of force is zero)}$$

In 2D, if $\vec{F} = F_x\hat{i} + F_y\hat{j}$:

$$\frac{\partial F_y}{\partial x} = \frac{\partial F_x}{\partial y}$$

This mathematical condition ensures path independence.

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3. Non-Conservative Forces - Detailed Theory

3.1 What is a Non-Conservative Force?

Definition: A force is said to be **non-conservative** if the work done by the force in moving an object from one point to another **depends on the path taken** between those points.

Alternative Definition: A force is non-conservative if the work done by it over a closed path is **not zero**.

3.2 Key Characteristics of Non-Conservative Forces

1. **Path Dependence**
   - Work done depends on the actual path followed
   - Different paths → different work done
   - Mathematical condition: $W_{AB}^{(path\ 1)} \neq W_{AB}^{(path\ 2)}$
2. **Closed Loop Property**
   - Work done in closed path is NOT zero
   - Mathematical condition: $\oint \vec{F} \cdot d\vec{r} \neq 0$
3. **Potential Energy Cannot Be Defined**
   - No unique potential energy function exists
   - Energy is dissipated (converted to heat, sound, deformation)
4. **Mechanical Energy is NOT Conserved**
   - Total mechanical energy decreases
   - Energy is converted to other forms (thermal, sound, etc.)
   - $\Delta E_{mechanical} = W_{non-conservative}$

3.3 Physical Meaning - Why Path Matters

Think of pushing a box across a room:

Friction Force (Non-Conservative):

- Take a straight path: friction opposes for short distance
- Take a zigzag path: friction opposes for longer distance
- Longer path → more work done against friction
- Work done: $W_f = -f \times d$ (where d is actual path length)
- Path matters significantly

Key Insight: Non-conservative forces dissipate energy. The longer the path, the more energy is lost.

3.4 Examples of Non-Conservative Forces

1. **Friction**
   - Kinetic friction: $f_k = \mu_k N$
   - Work done converts mechanical energy to heat
   - Always opposes motion
   - Work: $W_f = -f \cdot d$ (negative, energy dissipated)
2. **Air Resistance (Drag)**
   - Opposes motion through fluid

  - Converts kinetic energy to heat and turbulence
  - Depends on velocity: $F_d \propto v^2$ (at high speeds)
3. **Viscous Force**
  - Resistance in fluids
  - Energy dissipated as internal fluid motion and heat
  - Example: Object moving through honey or oil
4. **Tension in Cord (sometimes)**
  - Can be non-conservative depending on situation
  - If cord changes length or slides, work may be path-dependent
5. **Applied Force by Humans/Machines**
  - External forces that add or remove energy
  - Not inherent to system, so treated as non-conservative
6. **Normal Force (perpendicular)**
  - Generally does zero work (perpendicular to motion)
  - But can be non-conservative in certain scenarios

3.5 Energy Dissipation

Non-conservative forces convert mechanical energy into:

- **Heat** (most common with friction)
- **Sound** (vibrations, noise)
- **Deformation** (permanent shape change)
- **Light** (in some cases)

Important: Energy is not destroyed; it's transformed into forms we can't easily recover as mechanical energy.

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4. Comparison Table: Conservative vs Non-Conservative Forces

Property	Conservative Force	Non-Conservative Force
Path dependence	Independent of path	Depends on path taken
Work in closed loop	Zero ($\oint \vec{F} \cdot d\vec{r} = 0$)	Non-zero ($\oint \vec{F} \cdot d\vec{r} \neq 0$)
Potential energy	Can be defined uniquely	Cannot be defined
Mechanical energy	Conserved	Not conserved (dissipated)
Energy conversion	KE ↔ PE	Mechanical → Heat, sound, etc.
Examples	Gravity, spring force, electrostatic	Friction, air resistance, viscous drag
Work formula	$W = -\Delta U$	$W = -f \cdot d$ (path dependent)
Reversibility	Reversible process	Irreversible (energy lost)

Table 1: Comprehensive Comparison of Conservative and Non-Conservative Forces

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5. Work-Energy Theorem with Conservative and Non-Conservative Forces

5.1 General Work-Energy Theorem

The total work done by all forces equals the change in kinetic energy:

$$W_{total} = W_{conservative} + W_{non-conservative} = \Delta KE$$

5.2 When Only Conservative Forces Act

If only conservative forces do work:

$$W_c = \Delta KE$$

But $W_c = -\Delta PE$, so:

$$-\Delta PE = \Delta KE$$

$$\Delta KE + \Delta PE = 0$$

$$(KE_f + PE_f) - (KE_i + PE_i) = 0$$

$$E_f = E_i \quad \text{(Mechanical energy conserved)}$$

5.3 When Non-Conservative Forces Also Act

$$W_c + W_{nc} = \Delta KE$$

$$-\Delta PE + W_{nc} = \Delta KE$$

$$W_{nc} = \Delta KE + \Delta PE = \Delta E_{mechanical}$$

Key Equation:

$$W_{non-conservative} = \Delta E_{mechanical}$$

Interpretation:

- If $W_{nc} < 0$ (friction, air resistance): Mechanical energy decreases
- If $W_{nc} > 0$ (applied external force): Mechanical energy increases

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6. Key Formulas Summary

Concept	Formula	Meaning
Work by conservative force	$W_c = -\Delta U$	Negative of PE change
Closed path (conservative)	$\oint \vec{F}_c \cdot d\vec{r} = 0$	Zero work in closed loop
Mechanical energy conservation	$KE + PE = E = constant$	When only conservative forces
Work-energy with friction	$W_c + W_f = \Delta KE$	Total work = KE change
Friction work	$W_f = -f \cdot d = -\mu N \cdot d$	Always negative (opposes)
Change in mechanical energy	$\Delta E = W_{nc}$	Non-conservative work changes ME
Gravitational PE	$U_g = mgh$	Height-dependent
Spring PE	$U_s = \frac{1}{2}kx^2$	Displacement-dependent

Table 2: Essential Formulas for Conservative and Non-Conservative Forces

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7. Important Concepts

7.1 Why Can We Define Potential Energy for Conservative Forces?

- Work done is path-independent
- Depends only on position (not history)
- We can assign a unique energy value to each position
- This position-dependent energy is **potential energy**

Relation: $\vec{F} = -\nabla U = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right)$

7.2 Energy Flow in Real Systems

Ideal System (Only Conservative Forces):

$$\text{KE} \leftrightarrow \text{PE} \quad \text{(Energy cycles back and forth)}$$

Real System (With Non-Conservative Forces):

$$\text{Mechanical Energy} \rightarrow \text{Heat} + \text{Sound} + \text{Deformation} \quad \text{(One-way loss)}$$

7.3 Why Friction is Always Negative Work

Friction **always opposes motion**:

- Direction of friction: opposite to velocity
- Direction of displacement: along velocity
- Angle between $\vec{f}$ and $d\vec{r}$: 180°
- Work: $W = f \cdot d \cdot \cos(180^\circ) = -f \cdot d < 0$

7.4 Can a Force Be Both Conservative and Non-Conservative?

No. A force is intrinsically one or the other based on its fundamental nature:

- Gravity is always conservative
- Friction is always non-conservative
- The classification depends on the physical mechanism of the force

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8. Common Mistakes Students Make

Mistake 1: Assuming All Forces Conserve Mechanical Energy

Error: Thinking mechanical energy is always conserved.

Correction: Mechanical energy is conserved **only when** non-conservative forces (like friction) do zero work.

Mistake 2: Confusing "Conservative" with "Constant Force"

Error: Thinking conservative means the force is constant.

Correction:

- Conservative refers to **path independence**, not magnitude
- Spring force varies with position but is still conservative
- Gravity can vary with height (near Earth surface: constant; in space: varies) but remains conservative

Mistake 3: Forgetting Friction Does Negative Work

Error: Calculating friction work as positive.

Correction: Friction **always opposes motion**, so:

$$W_f = -\mu N \cdot d \quad \text{(always negative)}$$

Mistake 4: Path Independence Misunderstanding

Error: Thinking conservative forces don't do work.

Correction: Conservative forces DO work, but the work is the **same for all paths** between two points.

Mistake 5: Applying $W = Fd\cos\theta$ Without Considering Path

Error: Using simple $W = Fd$ formula for non-conservative forces without integrating over actual path.

Correction: For path-dependent forces, must integrate:

$$W = \int_{path} \vec{F} \cdot d\vec{r}$$

Mistake 6: Ignoring Direction of Non-Conservative Forces

Error: Calculating magnitude but forgetting friction opposes motion direction.

Correction: Always check direction:

- Friction: opposite to velocity/motion
- Air resistance: opposite to velocity
- Applied force: can be any direction (given in problem)

Mistake 7: Confusing Work by Gravity in Different Paths

Error: Thinking work by gravity depends on horizontal or slanted distance traveled.

Correction: Work by gravity depends **only on vertical displacement** Δh:

$$W_g = mg\Delta h$$

Horizontal motion: $W_g = 0$

Mistake 8: Not Recognizing When Mechanical Energy Changes

Error: Using conservation of mechanical energy when friction is present.

Correction: When friction acts:

$$E_f = E_i + W_{friction} \quad (\text{where } W_f < 0)$$

Mistake 9: Wrong Sign in Potential Energy Change

Error: Writing $W_c = \Delta U$ instead of $W_c = -\Delta U$.

Correction: Conservative force does **positive work** when PE **decreases**:

$$W_c = -\Delta U = -(U_f - U_i) = U_i - U_f$$

Mistake 10: Assuming Zero Work Means Zero Force

Error: Thinking if work is zero, force must be zero.

Correction: Work can be zero if:

- Force is zero, **OR**
- Force is perpendicular to displacement ($\cos 90^\circ = 0$), **OR**
- Displacement is zero
- Work in closed path by conservative force (returns to start)

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9. Multiple Choice Questions (MCQs)

Q1. Which of the following is a conservative force?

(A) Friction
(B) Air resistance
(C) Gravitational force
(D) Viscous drag

Answer: (C) Gravitational force

Explanation: Gravity is conservative because work done depends only on height difference, not path. Friction, air resistance, and viscous drag are all non-conservative.

Q2. The work done by a conservative force in a closed path is:

(A) Always positive
(B) Always negative
(C) Zero
(D) Depends on the path

Answer: (C) Zero

Explanation: By definition, $\oint \vec{F}_c \cdot d\vec{r} = 0$ for conservative forces.

Q3. A block slides down a rough inclined plane. Which forces are non-conservative?

(A) Gravity only
(B) Friction only
(C) Normal force only
(D) Both friction and normal force

Answer: (B) Friction only

Explanation: Gravity is conservative. Normal force does zero work (perpendicular). Friction is non-conservative and dissipates energy as heat.

Q4. If only conservative forces act on a system, then:

(A) Kinetic energy is constant
(B) Potential energy is constant
(C) Mechanical energy is constant
(D) Total energy decreases

Answer: (C) Mechanical energy is constant

Explanation: $E = KE + PE = constant$ when only conservative forces act. Individual KE and PE may change, but their sum remains constant.

Q5. A book is moved from point A to point B on a table with friction. The work done by friction:

(A) Is path-independent
(B) Depends on the path taken
(C) Is always zero
(D) Is positive

Answer: (B) Depends on the path taken

Explanation: Friction is non-conservative. Longer path means more distance, hence more (negative) work done by friction.

Q6. The potential energy can be defined for:

(A) All forces
(B) Only conservative forces
(C) Only non-conservative forces
(D) No forces

Answer: (B) Only conservative forces

Explanation: Potential energy exists only for conservative forces because work must be path-independent.

Q7. A ball is thrown upward and returns to the same height. If air resistance is present, then:

(A) Mechanical energy is conserved
(B) Mechanical energy decreases
(C) Mechanical energy increases

(D) Kinetic energy at return equals initial kinetic energy

Answer: (B) Mechanical energy decreases

Explanation: Air resistance is non-conservative. It dissipates mechanical energy as heat. The ball returns with less kinetic energy than it started with.

Q8. Work done by spring force in moving from extension x_1 to x_2 is:

(A) $\frac{1}{2}k(x_2^2 - x_1^2)$
(B) $-\frac{1}{2}k(x_2^2 - x_1^2)$
(C) $k(x_2 - x_1)$
(D) $-k(x_2 - x_1)$

Answer: (B) $-\frac{1}{2}k(x_2^2 - x_1^2)$

Explanation: $W_c = -\Delta U = -(U_f - U_i) = -\left[\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right]$

Q9. A block of mass 2 kg slides on a horizontal surface with initial velocity 10 m/s and comes to rest after traveling 20 m. The coefficient of friction is (take g = 10 m/s^2):

(A) 0.25
(B) 0.5
(C) 0.75
(D) 1.0

Answer: (A) 0.25

Solution:
By work-energy theorem: $W_f = \Delta KE$
$-\mu mg \cdot d = 0 - \frac{1}{2}mv_0^2$
$-\mu \times 2 \times 10 \times 20 = -\frac{1}{2} \times 2 \times (10)^2$
$-400\mu = -100$
$\mu = 0.25$

Q10. An object of mass 1 kg is lifted vertically upward through 5 m. Taking g = 10 m/s^2, the work done by gravity is:

(A) 50 J
(B) -50 J
(C) 0 J
(D) 25 J

Answer: (B) -50 J

Explanation: Gravity acts downward, displacement is upward (opposite direction).
$W_g = mg(-h) = 1 \times 10 \times (-5) = -50$ J

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10. Assertion and Reasoning Questions

Instructions: Each question contains an Assertion (A) and a Reason (R). Choose:

(a) Both A and R are true, and R is the correct explanation of A
(b) Both A and R are true, but R is NOT the correct explanation of A
(c) A is true, but R is false
(d) A is false, but R is true
(e) Both A and R are false

Q1.

Assertion (A): Gravitational force is a conservative force.

Reason (R): Work done by gravitational force depends only on initial and final positions, not on the path taken.

Answer: (a) Both A and R are true, and R is the correct explanation of A

Explanation: The reason directly explains why gravity is conservative—path independence is the defining property of conservative forces.

Q2.

Assertion (A): Friction always does negative work.

Reason (R): Friction always opposes the motion of objects.

Answer: (a) Both A and R are true, and R is the correct explanation of A

Explanation: Since friction opposes motion (R), the angle between friction force and displacement is 180°, making work negative (A). R explains A.

Q3.

Assertion (A): Potential energy can be defined for friction.

Reason (R): Friction is a non-conservative force.

Answer: (d) A is false, but R is true

Explanation: Potential energy cannot be defined for non-conservative forces. Friction is indeed non-conservative (R is true), which is why A is false.

Q4.

Assertion (A): The work done by conservative forces in a closed path is zero.

Reason (R): Conservative forces are path-independent.

Answer: (a) Both A and R are true, and R is the correct explanation of A

Explanation: Path independence (R) implies that going from A to B to A (closed path) yields zero net work (A).

Q5.

Assertion (A): Mechanical energy is not conserved when a ball bounces on the ground.

Reason (R): During collision, some energy is converted to heat and sound.

Answer: (a) Both A and R are true, and R is the correct explanation of A

Explanation: Non-conservative forces during impact dissipate energy (R), causing loss of mechanical energy (A).

Q6.

Assertion (A): Spring force is conservative.

Reason (R): Spring force is always constant.

Answer: (c) A is true, but R is false

Explanation: Spring force IS conservative (A is true), but it's NOT constant—it varies with displacement: $F = -kx$ (R is false).

Q7.

Assertion (A): Air resistance is a non-conservative force.

Reason (R): Work done by air resistance depends on the velocity of the object.

Answer: (b) Both A and R are true, but R is NOT the correct explanation of A

Explanation: Air resistance is non-conservative because work depends on path length (not explained by velocity dependence mentioned in R).

Q8.

Assertion (A): Normal force always does zero work.

Reason (R): Normal force is always perpendicular to the displacement.

Answer: (a) Both A and R are true, and R is the correct explanation of A

Explanation: When force is perpendicular to displacement, $\cos 90^\circ = 0$, so work is zero. R correctly explains A.

Q9.

Assertion (A): A block sliding on a frictionless incline conserves mechanical energy.

Reason (R): Only conservative forces (gravity) act on the block.

Answer: (a) Both A and R are true, and R is the correct explanation of A

Explanation: With no friction (only gravity acts), mechanical energy is conserved.

Q10.

Assertion (A): The total energy of the universe is conserved.

Reason (R): Non-conservative forces convert mechanical energy to other forms, not destroy it.

Answer: (b) Both A and R are true, but R is NOT the correct explanation of A

Explanation: Both are independently true statements about energy conservation at different levels, but R doesn't fully explain A (which is a broader principle).

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11. Practice Strategy and Tips

11.1 Identifying Force Type

Step-by-step approach:

1. **Read the problem carefully** - Identify all forces mentioned
2. **Check for dissipative forces** - Friction? Air resistance? → Non-conservative
3. **Identify field forces** - Gravity? Spring? Electric? → Usually conservative
4. **Determine if energy is lost** - Mentioned heat, sound, deformation? → Non-conservative present
5. **Check problem conditions** - "Frictionless," "smooth," "no air resistance" → Only conservative forces

11.2 Problem-Solving Strategy

For conservative force problems:

- Use conservation of mechanical energy: $E_i = E_f$
- Write: $KE_i + PE_i = KE_f + PE_f$
- Choose convenient reference level for PE (usually ground)

For non-conservative force problems:

- Use: $W_c + W_{nc} = \Delta KE$
- Or: $\Delta E_{mechanical} = W_{non-conservative}$
- Calculate friction work: $W_f = -\mu N \cdot d$ (negative!)

11.3 Common Problem Types

1. **Block on incline with friction** - Both conservative (gravity) and non-conservative (friction)
2. **Projectile with air resistance** - Gravity (conservative) + air resistance (non-conservative)
3. **Spring with damping** - Spring force (conservative) + damping (non-conservative)
4. **Vertical circular motion** - Gravity (conservative), sometimes friction at contact

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12. Summary and Key Takeaways

- **Conservative forces:** Path-independent, PE can be defined, mechanical energy conserved
- **Examples:** Gravity, spring force, electrostatic force
- **Mathematical test:** $\oint \vec{F}_c \cdot d\vec{r} = 0$ and $\nabla \times \vec{F} = 0$
- **Non-conservative forces:** Path-dependent, dissipate energy, ME not conserved
- **Examples:** Friction, air resistance, viscous drag
- **Effect:** Convert mechanical energy to heat, sound, deformation
- **Key equation:** $W_c = -\Delta U$ (conservative forces)
- **Key equation:** $\Delta E_{mechanical} = W_{non-conservative}$
- **Conservation of ME:** Only when non-conservative forces do zero work
- **Real systems:** Always have some non-conservative forces (friction, air resistance)

Master these concepts for NEET success!

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Understand the distinction between conservative and non-conservative forces to master energy conservation problems. Practice regularly and analyze each force in the system!
